Resistance to Collapse: The Tensor Interaction

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Abstract

We consider negative-parity excitations in ^{4}He, $J^\pi=0^-$ and 1^- with isospin $T=0$ and $T=1$. We show that, in contrast to one-particle one-hole results, in a shell model diagonalization none of the above collapses to zero energy as the strength of the tensor interaction is increased.

1　Introduction

Most of the studies of collapse of nucleon systems involve R.P.A. calculations. One can trace the oncoming of the collapse by gradually turning on an interaction to its full strength. Particle-hole excitations with certain quantum numbers which originally have a high excitation energy come down lower and lower in energy as the interaction strength is increased, and eventually they come to zero energy. At this point, it costs no energy to produce several of these particle-hole states. The nature of the ground state changes considerably.

Most of the interest in collapse has been for pion condensation. The particle-hole states of interest have quantum numbers S=1 T=1 and negative parity, e.g. 0^-, 1^- spin dipole, 2^- etc. With the interactions in current use, states with these quantum numbers are at high energies. Also, consistently corresponding states with isospin T=0 lie lower in energy than the T=1 states. But there is always the underlying possibility, especially for the spin excitations, that a stronger tensor interaction could help to bring down these modes in energy.

In this work, we will not deal directly with the problem of pion condensation but rather a related problem of increasing the overall strength of the tensor interaction. Rather than rely on R.P.A. calculations, we will perform the more accurate shell model diagonalization. We intend to show that the behavior of the energies of the 'particle-hole' states versus tensor strength is quite different in the shell model diagonalization than it is in the T.D.A. or R.P.A.

2　Method:

2 (a) Lowest Order

In lowest order, the configuration for the ground state of ^{4}He is $(0s)^4$, and

the excited states mentioned above are one-particle one-hole configurations. The 0^- state in lowest order has the unique configuration $(0p\frac{1}{2}\,0s\frac{1}{2}^{-1})$. The energy of this state is given by

$$E(0^-,T) = \epsilon_{0p_{\frac{1}{2}}} - \epsilon_{0s_{\frac{1}{2}}^{-1}} + \langle(0p_{\frac{1}{2}}0s_{\frac{1}{2}}^{-1})|V|(0p_{\frac{1}{2}}0s_{\frac{1}{2}}^{-1}\rangle^{0^-,T} \qquad (1)$$

where ϵ_i are the single-particle energies and the last term is the particle-hole interaction. It should be mentioned that we calculate the single-particle energies with the same interaction that is used to calculate the two-particle matrix elements. We used the schematic (or democratic) interaction described in a work by Zheng and Zamick [1]. It is of the form

$$V_{sche} = V_c + xV_{so} + yV_t \qquad (2)$$

where $c \equiv$ central, $s.o. \equiv$ spin-orbit, and $t \equiv$ tensor. For $x = 1$, $y = 1$ one gets a fairly good fit to the two-body matrix elements of more realistic interactions like Bonn A. We focus on the effects of the <u>tensor interaction</u> at the energies of the states in ^{4}He. We do this by varying y, the strength of the tensor interaction. In the simple one-particle one-hole picture, the single-particle energies do not depend on y (i.e. the first-order tensor contribution to these energies is zero) and only the particle-hole matrix element is affected. In the full shell model calculation the situation is more complicated-there are many configurations.

In a $1p - 1h$ calculation, the $J=0^-$ states have unique configurations $(0p_{\frac{1}{2}}0s_{\frac{1}{2}}^{-1})^{J=0,T}$ with $T=0$ or 1. Using equation (1), we note that as we increase y, the single-particle energies ϵ do not change. Obviously, the particle-hole interaction

$$V_{ph} = \langle(0p_{\frac{1}{2}}0s_{\frac{1}{2}}^{-1})|V|(0p_{\frac{1}{2}}0s_{\frac{1}{2}}^{-1}\rangle^{0^-,T}$$

will be linear in y. We find

$$V_{ph}(T = 0) = 2.575 - 3.820y \quad MeV$$

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$$V_{ph}(T = 1) = 3.445 - 1.270y \quad MeV$$

Note that the coefficient of y for T=1, i.e. the slope, is $\frac{1}{3}$ that for T=0. The excitation energy will decrease linearly in y and we clearly can get the 0^- states coming below the ground state by making y sufficiently large. The T=1 state in this model is always higher in energy than the T=0 J=0^- states. We have not performed R.P.A. calculations but we know from experience that when states come down in energy in T.D.A. calculations they come down even faster in R.P.A. calculations.

2 (b) Matrix Diagonalization

We perform a shell model matrix diagonalization, using the OXBASH code [2], for the J=0^+ ground state and excited J=0^- and 1^- states with both isospin T=0 and T=1 in ^{4}He. We allow the four nucleons to be anywhere in the first three major shells $0s$, $0p$ and $1s$-$0d$. We choose ^{4}He for practical reasons - it is easier to perform a high-quality shell model calculation in such a light nucleus.

In Table I we give the shell model results as a function of the strength parameter of the tensor interaction y (for x=1) for the excitation energies in ^{4}He of the lowest J=0^- and 1^- states with isospins T=0 and T=1. It should be mentioned that we have removed the 1^- T=0 spurious state- what is listed is the lowest non-spurious state. We also show the behaviour of the excitation energies of the J=0^- T=0 and T=1 states as a function of y in Fig 1.

For the J=0^- states, we see that both the T=0 and T=1 states come down in energy as we increase y up to a certain point (i.e. $y$$\simeq$ 3 for T=0) but then as y is further increased the excitation energy gets larger.

We note that the ground state binding energy changes as we increase y. In lowest order, i.e $(0s)^4$, there is no contribution to the binding energy due to the tensor interaction (this holds for any <u>major</u> shell. However, the nucleon-nucleon interaction induces configuration mixing into the ground-

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state wave function. For this more complicated ground state, the tensor interaction does contribute to the binding energy.

The change in binding energy in MeV of the ground state relative to the case $y{=}0$ is as follows:

$y{=}0$	0.00	$y{=}1$	1.83
$y{=}2$	7.01	$y{=}3$	14.68
$y{=}4$	23.99	$y{=}5$	34.42

we see that this change of energy starts out quadratic in y as we would expect from a second-order tensor effect.

Although the full matrix diagonalization does not lead to the negative parity excitations sinking below the ground state, the ground state wave function does change as the tensor strength y is increased. The occupancy of shells higher than $0s$ increases with increasing y. From $y = 0$ to $y = 5$, $0s$ occupancies are 3.53, 3.46, 3.28, 3.08, 2.91 and 2.76. The corresponding $0p_{\frac{1}{2}}$ occupancies are 0.08, 0.17, 0.38, 0.61, 0.80 and 0.95. Hence the nature of the ground state does change but it does so in a continuous manner.

3   Closing Remarks:

Whereas simple $1p-1h$ calculations with very strong tensor interactions can yield a collapse of negative parity excitations, we find in superior full shell model calculations this is not the case. We will not here go into a detailed analysis of why this is so. Suffice to say that there are several second and higher order corrections beyond those of the T.D.A. or R.P.A. First of all the single particle energies get renormalized in second order [3] [4]. Second the particle-hole interaction can get renormalized e.g. by the exchange of a phonon between the particle and the hole [5]. The model we have presented here is not quite the same as that of pion condensation. For example, we

have multiplied the entire tensor interaction by a constant y- this in effect contains effects of both the ρ and π mesons. However, we feel that we have made an important point- that many more effects than those contained in $1p - 1h$ T.D.A. or R.P.A. calculations must be taken into account.

Acknowledgment

References

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Table I. The excitation energies of the lowest-lying negative parity states in ^{4}He as a function of the strength of the tensor interaction y.

y	$J=0^-$ $T=0$	$J=0^-$ $T=1$	$J=1^-$ $T=0$	$J=1^-$ $T=1$
0.	20.88	22.80	19.98	21.35
1.	18.78	22.06	21.75	22.16
2.	17.41	22.40	24.79	23.70
3.	17.04	24.09	28.98	25.95
4.	17.51	26.77	33.88	28.79
5.	18.58	30.13	39.20	32.07
10.	28.03	51.97	67.27	51.59

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